Importance of Nonlinear and Multivariable Flexibility Coefficients in the Prediction of Human Cervical Spine Motion

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The flexibility matrix currently forms the basis for multibody dynamics models of cervical spine motion. While studies have aimed to determine cervical motion segment behavior, their accuracy and utility have been limited by both experimental and analytical assumptions. Flexibility terms have been primarily represented as constants despite the spine's nonlinear stiffening response. Also, nondiagonal terms, describing coupled motions, of the matrices are often omitted. Currently, no study validates the flexibility approach for predicting vertebral motions; nor have the effects of matrix approximations and simplifications been quantified. Therefore, the purpose of this study is to quantify flexibility relationships for cervical motion segments, examine the importance of nonlinear components of the flexibility matrix, and determine the extent to which multivariable relationships may alter motion prediction. To that end, using unembalmed human cervical spine motion segments, a full battery of flexibility tests were performed for a neutral orientation and also following an axial pretorque. Primary and coupled matrix components were described using linear and piecewise nonlinear incremental constants. A third matrix approach utilized multivariable incremental relationships. Measured motions were predicted using structural flexibility methods and evaluated using RMS error between predicted and measured responses. A full set of flexibility relationships describe primary and coupled motions for C3-C4 and C5-C6. A flexibility matrix using piecewise incremental responses offers improved predictions over one using linear methods ($p<0.01$). However, no significant improvement is obtained using nonlinear terms represented by a multivariable functional approach ($p<0.2$). Based on these findings, it is suggested that a multivariable approach for flexibility is more demanding experimentally and analytically while not offering improved motion prediction. [DOI: 10.1115/1.1504098]

Keywords: Flexibility, Cervical, Model, Matrix, Motion Segment

Introduction

Components of flexibility and stiffness matrices have been widely used for characterization of the motions of the cervical spine as a whole and its individual motion segments [1,2]. Owing in part to an absence of published biomechanical data, many different models have simplified the coefficients of the matrices used to predict spinal motions and loads [1–5]. However, the accuracy and appropriateness of these simplifications has not been addressed.

Spinal flexibility and stiffness coefficients are often set equal to zero or equated to each other, based on conservation of energy and the assumption of anatomical sagittal plane symmetry of the spine [1–5]. In this way, the computational demands of the mathematical analysis are simplified. Experimental studies characterize the mechanical properties of the neck and spinal motion segments [1,6–10]; however, reported data have historically been limited to linear constants or univariable nonlinear functions. In volunteer studies of combined loading, human neck torques have been expressed as functions of the primary rotations only [5,11–13]. As such, loads developed as a result of coupled displacement are falsely assigned to the primary term of the matrix. While many experimental studies have been performed to characterize the spine's mechanical properties in response to loading out of the sagittal plane [4,6,8,14–17], few of these studies describe the behavior specifically for the cervical motion segment and the matrix terms are reported as constants at a single prescribed load, reflecting linear relationships. Despite experimental efforts to fully describe the cervical spine's flexibility [18], there remains absent in the literature an experimental study reporting accurate matrix terms describing all motions (both in and out of the plane of loading), specifically for the cervical spine.

The spine's nonlinear responses suggest that matrix component definitions should be nonlinear [1,9]. Flexibility also changes with reversals of the load direction: flexibility in flexion differs from extension, as do the responses in compression and tension [1–4]. These differences suggest that nondiagonal terms actually may not be assumed to be equal despite an approximately midsagittally symmetric structural geometry. This nonlinearity of the flexibility coefficients further suggests that each term may be a nonlinear function of loading in each of the six degrees of freedom. Therefore, motion in any given direction, x_1, is dependent on the loads and flexibilities in all degrees of freedom (Eq. 1):

$$x_1 = f_{11}F_1 + f_{12}F_2 + f_{13}F_3 + f_{14}F_4 + f_{15}F_5 + f_{16}F_6, \qquad (1)$$

where each of the flexibilities (f) are themselves multivariable functions of state of the loads (F_i) in every direction. Complete determination of the multivariable relationships for each coefficient in the flexibility matrix, therefore, requires measuring the relationship between displacements and forces for every possible

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loading state ad infinitum. No study has examined the extent to which matrix coefficients vary with each possible loading state. That is, no investigation has implemented and validated a multivariable nonlinear motion segment flexibility matrix model for the cervical spine.

It is the purpose of this study to examine the importance of the nonlinear components of the cervical spinal motion segment flexibility response. It is hypothesized that incorporation of nonlinear motion segment behavior in a flexibility matrix model will provide improved motion prediction over a linear approach. In testing this hypothesis, a quantitative assessment is provided of the extent to which the multivariable functional terms of the flexibility matrix can be simplified while still providing useful predictive ability of motion segment behavior.

Methods

Unembalmed human cervical C3-C4 and C5-C6 motion segments from six donors (Table 1) were each cast into aluminum cups using a reinforced polymethylmethacrylate resin. The neutral anatomic position was obtained by casting C3-C4 at 9° and C5-C6 and 17° relative to horizontal. An array of three stainless steel wires terminating in opaque black nylon spheres (7.94 mm diameter) was inserted in the anterior portion of each vertebral body for tracking joint motions. A single black sphere was fastened to the midpoint of the anterior surface of each vertebral body and served as the origin of the local coordinate system (Fig. 1).

A flexibility frame applied pure bending moments or pure forces to the superior vertebra of each motion segment, while the inferior vertebra was rigidly attached to the base of the test frame (Fig. 2). A counterweight applied a vertical load to balance the weight of the loading apparatus. A six-axis load cell (GSE Incorporated, Farmington Hills, MI) beneath the specimen measured forces. Pure moments were applied by a force couple. Translational forces were applied through the center of the superior vertebral body. The flexibility frame was designed to allow visualization of the motion tracking arrays using two CCD cameras (Kodak EM-2, Eastman Kodak, Charlotte, NC) oriented approximately 60° apart, each with a resolution of 119×192 pixels. The stereoimaging camera system and load cell recordings were synchronized to ensure temporal registration of vertebral body motion and load cell data.

Prior to flexibility testing, specimens were preconditioned by applying 30 cycles of flexion-extension moment. Moments and translational forces were applied quasistatically in all three planes: sagittal, coronal and transverse. Moment flexibility tests were applied in 0.15 Nm increments up to 1.05 Nm, followed by 0.30 Nm increments up to 2.55 Nm. Force flexibility tests were applied in 4 N increments up to 20 N, followed by increments of 8 N up to 60 N. At each applied load, 30 seconds of creep time were allowed before data acquisition. Flexibility testing was performed first in the sagittal plane followed by the coronal and transverse. Based on the anatomical sagittal plane symmetry of cervical motion segments, lateral bending moments, lateral shear, and axial torsion were assumed to produce symmetric vertebral motions. Therefore, each specimen was randomized for mechanical testing in either the right or left direction only, with the same direction applied for all lateral loading.

Matched flexibility testing was performed following an axial torque (pretorque). An axial pretorque of 1 Nm moment in the counterclockwise ($-\theta_z$) direction (Fig. 1) was applied and the complete flexibility testing was repeated. This preload was approximately 5% of the failure torque for these cervical levels and was expected to produce between 2–3 degrees of axial rotation [7]. The pretorque flexibility test battery was the same as that for the neutral configuration described above. However, because of the loss of sagittal plane symmetry with a pretorque, both the right and left directions of loading were tested in the presence of the pretorque.

For validation of model performance, a combined load was applied to each specimen to provide data. Forces were applied along the vertical direction using a pulley with an oblique orientation relative to the sagittal and coronal planes to create a com-

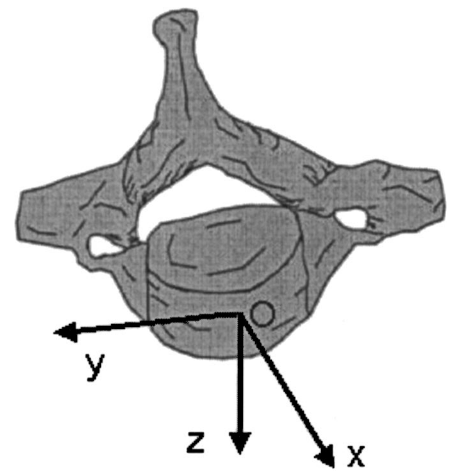

Fig. 1 Schematic showing the local coordinate system of a vertebra, with its origin (O) at the center of the anterior face. Also illustrated are the axes orientations (x,y,z) for the local coordinate system. As shown, the positive X-axis is oriented anteriorly along the anterior-posterior direction.

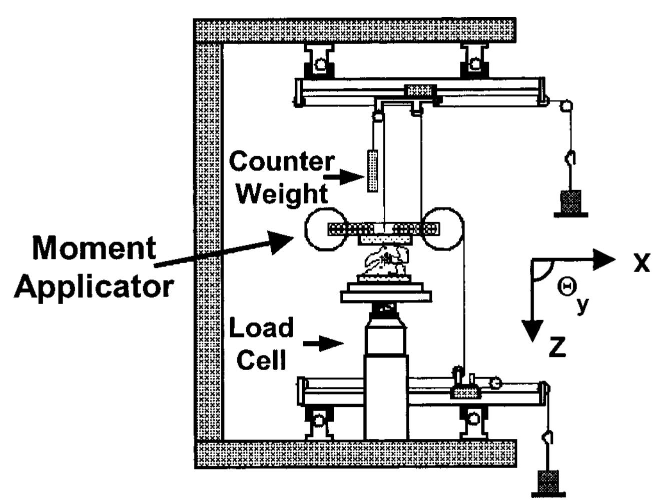

Fig. 2 Schematic diagram showing the flexibility frame with bending moment applicator. The test frame is equipped with a load cell, moment and force applicators, and stereoimaging cameras for motion tracking. Also shown in this illustration is the orientation of the global coordinate system.

Table 1 Specimen donor sex, age, height and weight.

Specimen ID	Cervical Level	Age/Sex	Height (cm)	Weight (kg)
A3	C3-C4	71/M	188	90
A5	C5-C6	71/M	188	90
B3	C3-C4	71/M	178	85
B5	C5-C6	71/M	178	85
C3	C3-C4	52/M	N/A	54
C5	C5-C6	52/M	N/A	54
D3	C3-C4	68/M	178	54
D5	C5-C6	68/M	178	54
E3	C3-C4	66/M	183	117
E5	C5-C6	66/M	183	117
F3	C3-C4	60/M	180	117
F5	C5-C6	60/M	180	117

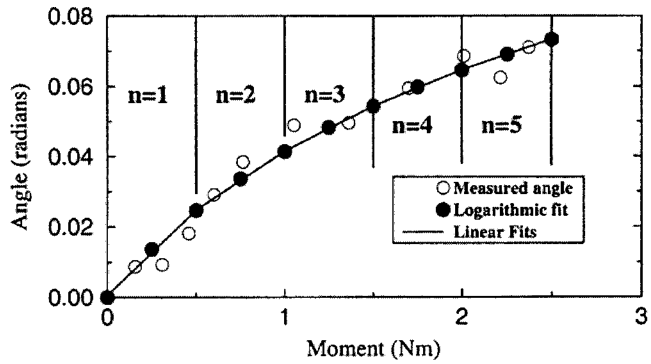

Fig. 3 Shown is a representative nonlinear approximation (filled circles) of the measured (open circles) response curve for a typical specimen, illustrating the use of five piecewise linear functions to characterize the nonlinear response. Also shown in this plot is the closeness of the linear fits to the logarithmic description of the response.

bined compression-flexion-lateral-bending load. Loads were applied in 1.9 N increments, with 30 seconds of creep time at each step. For validation loading, maximum applied moments were 3.5 Nm in flexion and 1.25 Nm in lateral bending. The load data were measured using the six axis load cell and used as input to the flexibility matrix. Vertebral motions were also recorded to validate the model outputs.

Marker positions were digitized from the stereoimages and processed using Direct Linear Transformation (DLT) [19–21]. Relative incremental translational and rotational motions of the intervertebral joint specimens were calculated using the vertebral origin position and Euler angle decomposition [22], respectively. The order of decomposition of rotations was flexion-extension, axial torsion, and lateral bending. Translational displacements and Euler angles were reported as the motions of the body fixed coordinate system of the superior vertebra relative to the spatially fixed coordinate system of the inferior vertebra. Images were combined with the force data for each loading increment, providing primary and coupled motion segment flexibility response relationships. Errors were determined for this system by comparing known measurements, using a motion application jig, RVDT, and LVDT, with those calculated using stereophotogrammetry techniques. There errors include those associated with image resolution, digitization, DLT, and decomposition. The mean error calculated for angular rotation for this study was $0.1\pm0.07°$. Similarly, the mean error for the calculated translations from digitization and DLT for this system was 0.15 ± 0.34 mm.

Each response curve (both primary and coupled) derived from the flexibility testing of neutral specimens was fit up to 2.5 Nm of applied bending moment and 50 N of applied translational force. Curve-fitting used the nonlinear logarithmic function:

$$x_i = A_{ji}\ln(B_{ji}F_j+1) \quad \text{(not summed)}, \tag{2}$$

where A and B are constants and x_i and F_j are the corresponding displacement and force components. This function provides a good description of biologic tissue behavior [1,23–25]. A complementary set of functions was also compiled for flexibility testing following an axial pretorque.

Flexibility matrix components were assigned according to the following methods. Each of the motion response logarithmic curve fits defined by Eq. 2 was simplified using piecewise linear functions over five regions (with increments of 0.5 Nm or 10 N) (Fig. 3). In each region, a slope was defined. Assignment of the appropriate component term of the flexibility matrix was determined by the magnitude of the imposed loads and the specific loading region ($n=1$–5) in which it was contained (Fig. 3).

For each load step of the validation test, an incremental flexibility matrix was constructed using a look-up table approach and the regional slopes for the motion response curves described above. For a given increment of load, the direction and magnitude of the load was determined for each component and the appropriate flexibility term (slope) was inserted into the flexibility matrix. At each increment of validation loading, this process was used for each of the six components of load to fill all of the terms of the matrix. Using each incremental flexibility matrix, the six incremental displacement components, (δx_i) were calculated for each applied load by multiplying the incremental flexibility matrix ([f]) by the incremental force vector ([δF]):

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} & f_{26} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} \end{bmatrix} \begin{bmatrix} \delta F_1 \\ \delta F_2 \\ \delta F_3 \\ \delta F_4 \\ \delta F_5 \\ \delta F_6 \end{bmatrix} = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \\ \delta x_5 \\ \delta x_6 \end{bmatrix} \tag{3}$$

Displacement components (x_i) for a given F_j were calculated by adding the predicted incremental motions (δx_i) to the displacements determined from the prior load step. In this way, displacements were calculated by stepping through each incrementally assembled flexibility matrix.

Three approaches for representing the full flexibility matrix were developed to assess the importance of nonlinear and multivariable behavior of the matrix. The *linear* model represented each flexibility matrix term as one linear constant, f_{ij}, over the entire range of loading. The *piecewise* nonlinear model represented each flexibility matrix term, $f_{ij,n}$, as an incremental linear constant based on the slope of the logarithmic fit relationships in the appropriate region (Figure 3, $n=1$–5). The *multivariable* model used the piecewise nonlinear method with the addition that each coefficient of the flexibility matrix was a multivariable function of the imposed load, F_j and the imposed axial torque, M_z. Thus, for an applied load in any direction, the flexibility coefficient in any region, $f_{ij,n}(F_j, M_z)$, was determined by linearly interpolating between the flexibility data derived from the neutral and pretorqued flexibility tests. This approach was used to assign the flexibility terms for each direction except that of axial torsion. Axial torsion flexibility terms were expressed only as functions of the torsional load, M_3, because the neutral flexibility fully describes the specimen behavior in this direction.

Model performance was assessed for each specimen using the combined loading validation experiments. Loads measured during these experiments provided the input to each of the three flexibility matrix representations (*linear, piecewise, multivariable*). The measured vertebral motions were compared to the matrix-predicted motion. Predictive ability was assessed using the root mean square (RMS) error:

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i^{\mathrm{mes}} - x_i^{\mathrm{pred}}\right)^2}, \tag{4}$$

where x_i^{mes} denotes the experimentally measured displacement component and x_i^{pred} denotes the corresponding predicted displacement component at each load increment, *i*. For each specimen, RMS error was calculated for each displacement and rotation. A one-way ANOVA test ($F>4.10$, $p<0.05$) was used to compare RMS error differences for the *linear, piecewise*, and *multivariable* models. Multiple comparison testing was performed using a Tukey's test to identify significant differences between the three models, at a p-value below 0.05.

Results

Flexibility data were well represented by the logarithmic functions (Fig. 3, Eq. 2). These logarithmic curve-fits accounted for 97.1±15.4% of the variance in the raw data. For each specimen tested, all flexibility responses, describing primary and coupled motions, were fit by these functions. Incremental data along these individual fits were then used to construct an average set of responses for each of the C3-C4 and C5-C6 motion segment levels. The modeling constants, *A* and *B*, in Table 2 indicate each of the coefficients describing the average fits for the data. For each of the twelve motion segments, fifty-four response curves were determined for the nine directions of loading in the neutral configuration; and for each direction of applied load, six relationships described the primary and five coupled motions as functions of the imposed load (Figs. 4(*a*, *b*)).

The average applied pretorque for specimens was 0.92±0.07 Nm, producing 2.4±1.1° of axial rotation of the superior vertebra. Pretorque resulted in 2.6±1.3° of rotation for C3-C4 which was not significantly different than the 2.2±0.8° for C5-C6 ($p=0.18$). Sagittal plane loading gave rise to primarily sagittal plane motion. For the neutral test configuration, the out-of-plane coupled loads and motions were small (<6%) compared to the main primary and coupled motions for imposed loading both in the sagittal and coronal planes (Figs. 4(*a*, *b*)). However, following a pretorque, anatomic symmetry was absent and larger out-of-plane motions occurred (~50% of primary motions).

Prediction of angular and translational displacement differed significantly among the models (Figs. 5, 6, Table 3). The flexion angle was the best predicted of all components. Using the *linear* model, the RMS for predicted flexion angles was 58% (mean RMS value of 0.069±0.04 radians) of the average full range of rotation (0.119±0.04 radians) in this direction (Fig. 7). However, this error was reduced to 23% of the full rotation using the *piecewise* model (mean RMS of 0.027±0.03 radians). Yet, for lateral bending and axial torsion components, the RMS errors, while significantly decreased in some cases, were never reduced to less than 74% of the average full scale rotational component in each of these directions (lateral bending 0.078±0.05 radians; torsion 0.051±0.04 radians).

Translation predictions were more poorly predicted than rotations using these matrix approaches (Figs. 5, 6). For anterior (x) translation, RMS errors were nearly four times the magnitude of the measured translation in this direction. RMS errors in this direction ranged from 16.25 to 16.65 mm for the various model approaches. In contrast, average translations for the generalized experimental loading scenario were small: 3.8±2.8 mm in anterior (x) translation, 4.6±5.3 mm in inferior (z) translation, and 1.8±1.9 mm for the lateral (y) direction.

The *piecewise* model performed better than the *linear* model over the entire range of loading (Figs. 5, 6). Use of piecewise linear responses resulted in decreased RMS error for all of the angular measurements (Figures 5–7, Table 3). Errors in prediction of the combined loading test were reduced for flexion by 61% ($p=0.007$) and for lateral bending by 34% ($p=0.01$) (Fig. 7). However, while the mean RMS error for axial rotation was decreased from 0.057±0.04 radians with the *linear* model to 0.044 ±0.03 radians for the *piecewise* model, the difference was not statistically significant ($p=0.081$). The lateral (y) and inferior (z) translation errors were decreased using a piecewise linear approach, yet not significantly ($p>0.06$). The anterior (x) translations were not different for the *linear* (mean RMS=16.25 mm) and *piecewise* (mean RMS=16.65 mm) models ($p=0.18$).

The *multivariable* matrix model predicted rotations with lower RMS errors than the *linear* model but not significantly different than the *piecewise* model (Table 3). Angular predictions were significantly improved when compared to the *linear* model: flexion ($p=0.006$), lateral bending ($p=0.015$), axial torsion ($p=0.045$). Prediction of translations was not significantly changed ($p>0.1$), with the *linear* model having RMS errors of 16.25±45.07, 6.53 ±5.78 and 2.54±1.63 mm and the *multivariable* model predicting translations with errors of 16.51±44.78, 5.58±5.28, and 2.22 ±1.43 mm, for the anterior (x), inferior (z) and lateral (y) trans-

Table 2 Summary of Modeling Constants for Logarithmic Fits of Average Flexibility Responses: For each motion segment level (C3-C4, C5-C6) an average response was determined for all specimens and this response was fit with a logarithmic function (Eq. 2). The constants, A and B, for these average fits are provided for the flexibility relationships (f_{ii}) as shown below. Where applicable, positive and negative relationships were determined. Of note when using these average fits to the data, angular rotations are expressed in radians.

Matrix Term	C3-C4		C5-C6		Matrix Term	C3-C4		C5-C6	
	A	B	A	B		A	B	A	B
$+f_{11}$	1.06	1.33	0.776	0.273	$+f_{41}$	4.08	0.059	−0.005	27144
$-f_{11}$	−2.12	−0.04	−1.13	−0.069	$-f_{41}$	−0.066	−0.826	0.231	−0.032
$+f_{12}$	−0.275	4.72	24.18	0.00004	$+f_{42}$	3.54	0.019	0.106	0.172
$-f_{12}$	−0.948	0.012	0.813	−0.091	$-f_{42}$	0.228	0.014	−4.01	0.007
$+f_{13}$	−0.985	1.79	−1.08	0.640	$+f_{43}$	−0.072	−0.378	0.033	45.51
$-f_{13}$	0.863	−2.17	2.38	−0.241	$-f_{43}$	0.290	−0.684	−0.520	0.198
$+f_{14}$	0.943	0.028	−0.001	−0.020	$+f_{44}$	1.26	0.090	6.786	0.0128
$+f_{15}$	0.474	7.31	0.433	0.876	$+f_{45}$	0.530	3.76	0.138	7.19
$+f_{16}$	0.077	7.38	0.134	12.46	$+f_{46}$	−0.146	−0.340	0.224	6.99
$+f_{21}$	3.15	0.218	1.10	0.040	$+f_{51}$	−0.033	0.351	0.0001	439.5
$-f_{21}$	−0.863	−0.060	−0.212	−0.808	$-f_{51}$	23.35	0.00002	0.001	−992.9
$+f_{22}$	0.293	0.188	0.203	0.694	$+f_{52}$	0.006	0.321	−0.0001	889.2
$-f_{22}$	−0.148	−21.41	−0.674	−1.38	$-f_{52}$	−68.26	0.00001	−0.001	−1.20
$+f_{23}$	−3.78	0.467	−0.131	9.21	$+f_{53}$	−0.273	−0.089	0.0324	0.159
$-f_{23}$	1.81	−1.49	0.030	−24.97	$-f_{53}$	0.037	−0.472	5.32	−0.00001
$+f_{24}$	1.66	0.089	−0.011	6236	$+f_{54}$	−0.021	0.031	−0.552	−0.00001
$+f_{25}$	1.09	3.84	0.254	30.82	$+f_{55}$	0.060	0.818	0.0265	3.716
$+f_{26}$	1.64	1.40	0.833	0.533	$+f_{56}$	0.012	9.03	0.0348	0.731
$+f_{31}$	−0.050	1.55	−0.0431	0.177	$+f_{61}$	0.019	−0.013	−0.008	0.0381
$-f_{31}$	0.037	0.087	0.0443	−0.0491	$-f_{61}$	−0.051	−0.003	0.002	0.275
$+f_{32}$	0.009	0.318	0.023	0.114	$+f_{62}$	−0.004	0.581	−0.001	0.027
$-f_{32}$	−0.452	−0.006	−0.0362	−0.542	$-f_{62}$	−0.003	−0.158	−0.001	−10.07
$+f_{33}$	0.090	0.573	0.0568	1.377	$+f_{63}$	−0.027	0.341	0.003	5.94
$-f_{33}$	−0.070	−1.87	−0.556	−1.646	$-f_{63}$	0.003	−5.56	−0.004	−6.92
$+f_{34}$	−0.012	−0.013	−0.001	163.1	$+f_{64}$	0.014	0.139	0.132	0.126
$+f_{35}$	−0.013	3.06	−0.023	1.47	$+f_{65}$	0.013	5.63	0.011	3.33
$+f_{36}$	−0.0004	279769	−0.009	0.085	$+f_{66}$	0.041	2.21	0.058	0.882

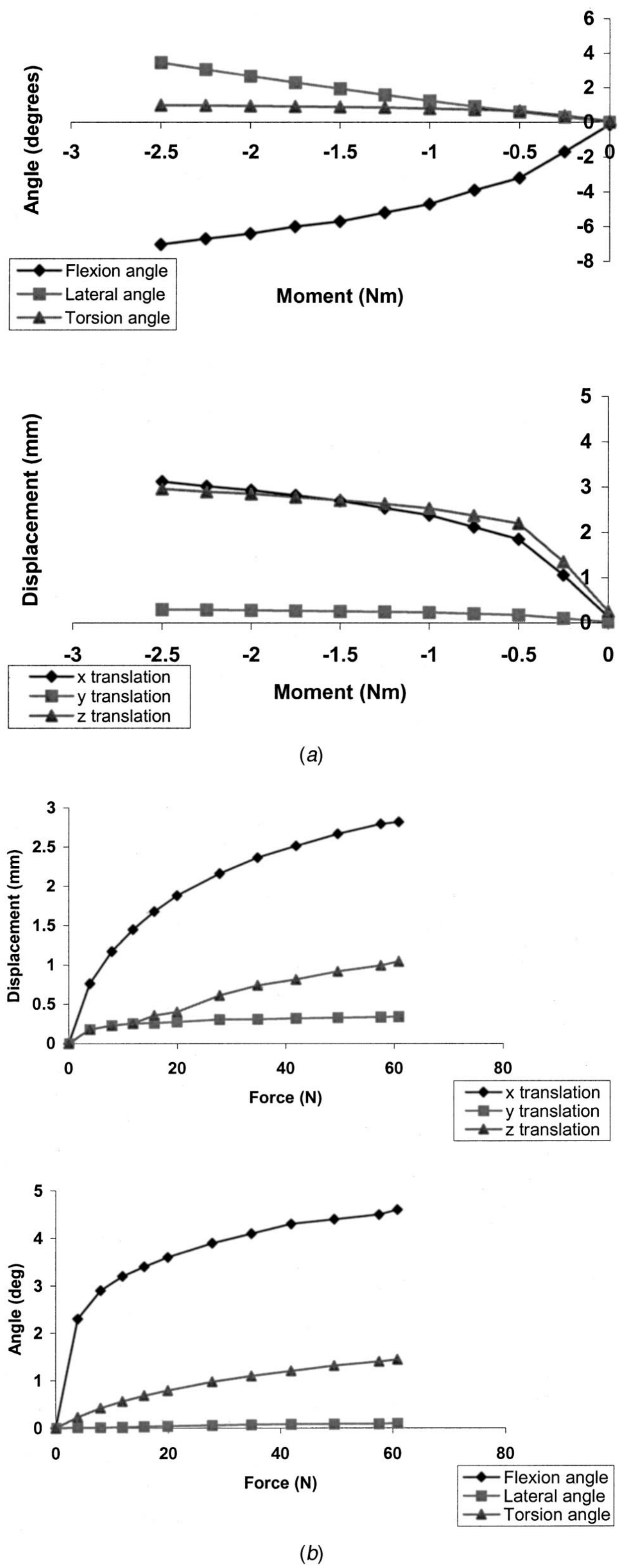

Fig. 4 (*a*) Shown are the primary (flexion angle) and coupled motion responses for an imposed flexion moment on a neutral specimen. The upper plot shows the rotational components and the lower plot demonstrates the coupled translations. These results illustrate the effects of loading in the sagittal plane, producing small lateral translations (y) and bending and axial torsion rotations. (*b*) Primary and coupled motion response magnitudes for a posteroanterior (+x) shear force applied to a representative neutral specimen.

lations, respectively. However, there was no significant difference in performance between the *piecewise* and *multivariable* nonlinear models for prediction of any of the components of displacement ($p>0.2$) (Table 3).

Discussion

A wide range of modeling efforts rely on the matrix approach for describing structural responses. Lacking in the literature is a study investigating the influence of these assumptions on the responses of the cervical spine in combined loading. Therefore, the goal of this study was to investigate the benefits of using linear, piecewise univariable nonlinear, and piecewise multivariable nonlinear matrix representations of cervical spinal motion segment flexibility, assessing flexibility model performance for predicting motions in combined compression-flexion-lateral bending loading.

While experimental studies have reported spinal flexibility and stiffness parameters [1,2,6,8,9,16,26], data have commonly been based on the assumptions that linear constants and primary terms can provide adequate descriptions of motion segment responses. Linear constants approximating the nonlinear matrix terms have been reported without justification for such an approach [4,8,9,14,17]. To date there has been no study that has compared model performance using linear matrix methods to those of piecewise nonlinear methods. Moreover, detailed flexibility descriptions for specific cervical levels have been absent in the literature, requiring scaling factors to approximate the cervical motion segment flexibility based on thoracic data [2–4].

The logarithmic fit used herein provided the information necessary to describe the average flexibility of six specimens at each cervical level and have good agreement with other studies in the literature. Calculating a flexibility from studies of the C5-C6 joint reported by Goel et al. [27], and comparing each flexibility term to an approximate linear flexibility term in this study as determined by the average constants in Table 3, indicates agreement within 10 and 30% for extension, lateral bending, and axial torsion. Shea et al. [9] reported constant flexibility terms describing translational and rotational loading in the sagittal plane, for specimens in mid (C2-C5) and lower (C5-T1) cervical regions. Considering the effects of multisegmental specimens and a linear contribution to flexibility, the flexibility terms from our data at the loads specified in the study by Shea et al. [9] are in agreement for both regional levels, with the largest difference between the two studies in the posterior shear direction (13% difference between studies). Limited experimental and computational work has examined the effect a preload would have on altering flexibility. Shea et al. [9] reported increased flexibility in extension following 10–16° of prerotation in specimens of two cervical joints. No data were provided for flexion loading, yet failure loads decreased for flexion following a pretorque compared to failure for the neutral specimens. Similarly, Yang et al. [28] reported increased flexibility in posterior shear for cervical spinal segments in the presence of an axial compressive preload.

While a nonlinear description of the mechanical response of vertebral joints has been discussed in a few experimental studies, implementation of this approach has been limited to primary motion in flexion-extension bending [1,9]. The piecewise nonlinear representations of the flexibility matrices in this study significantly reduce the error and improve predictive ability compared to the *linear* model. Despite the common use of constant flexibility terms in the literature, errors in predicted rotations are decreased using even simple piecewise nonlinear terms to incrementally describe the nonlinear nature of the motion responses. Moreover, such a *piecewise* linear approach can be easily implemented in viscoelastic and dynamic applications.

In theory, the nonlinearity of the flexibility response suggests that spinal flexibility is a function of its position in all six degrees of freedom. If true, three-dimensional motion behavior needs to

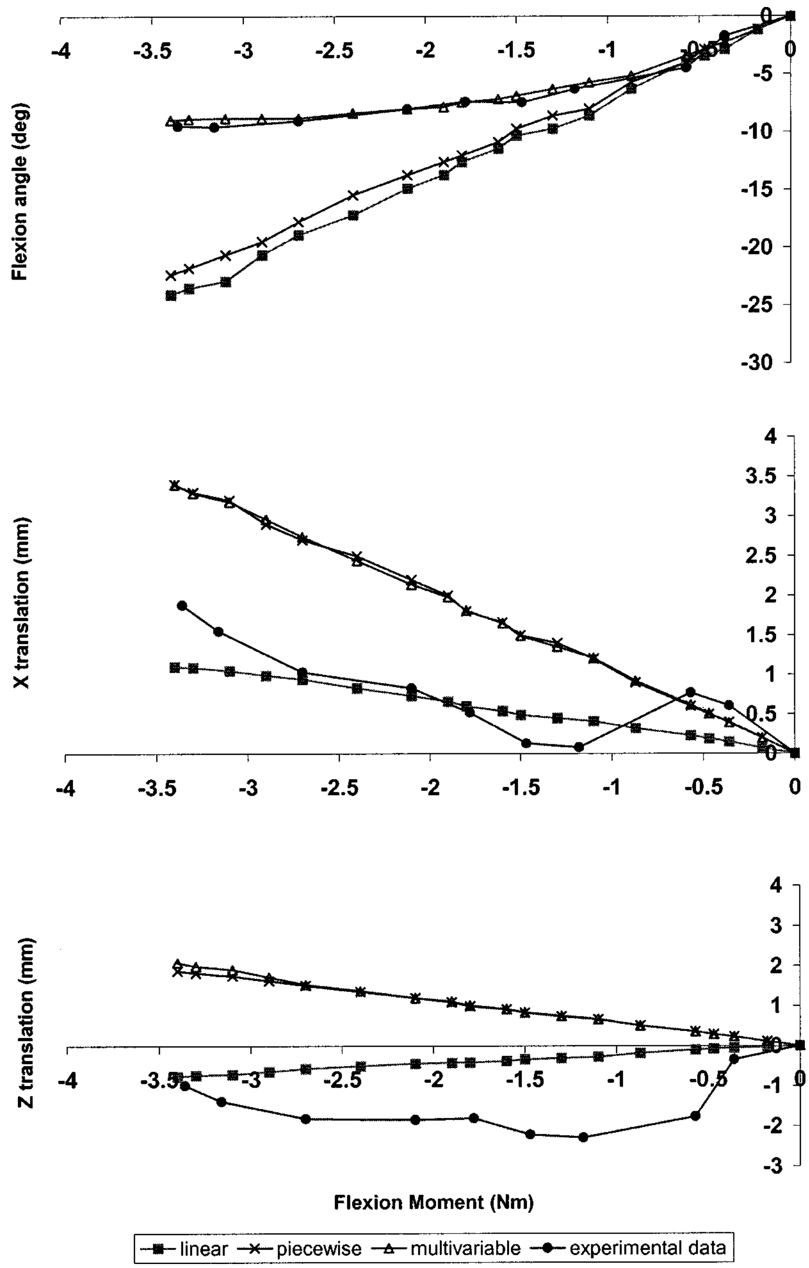

Fig. 5 Sagittal plane displacement components for a typical specimen. The sagittal plane experimental validation data, as well as the predicted motions from the three matrix models, are shown. For flexion in particular, the improved performance of the *multivariable* model is evident over the *linear* and *piecewise* models. For simplicity, the sagittal plane components of motion are shown as a function of applied flexion moment.

be characterized at each possible initial position of the spine. To assess the significance of this theory, the flexibility testing protocol with an axial pretorque investigated the benefit of incorporating segmental responses as functions of multiaxial loads at differing initial positions. While increased performance was observed through reduction in the angular rotation errors predicted by this model, the lowered errors were not significantly different from those of the *piecewise* model derived from neutral flexibility testing (Table 3, Fig. 7). While spinal flexibility may indeed be a function of its three-dimensional positioning and all of the loads acting on it in any given configuration, a model incorporating just one of these additional load directions requires twice the necessary experimental data without significant improvement in predictive ability over a model using univariable mechanical testing. This suggests that the possible enhanced performance of multivariable flexibility matrix terms does not justify the experimental overhead it creates, whereas a piecewise nonlinear model which includes all primary and coupled terms is both justified and experimentally tractable.

The piecewise models (*piecewise* & *multivariable*) performed with increased predictive ability and decreased RMS errors (Fig. 7). While improvements were most dramatic for flexion, they were not as prominent for lateral bending and torsion components, suggesting a difficulty in predicting these two very highly coupled motion components. Also, at these applied load levels, the motion segment flexibility in these out-of-plane directions is lower than its flexibility in flexion. Using a single constant term, model prediction is highly sensitive to the range of loading from the flex-

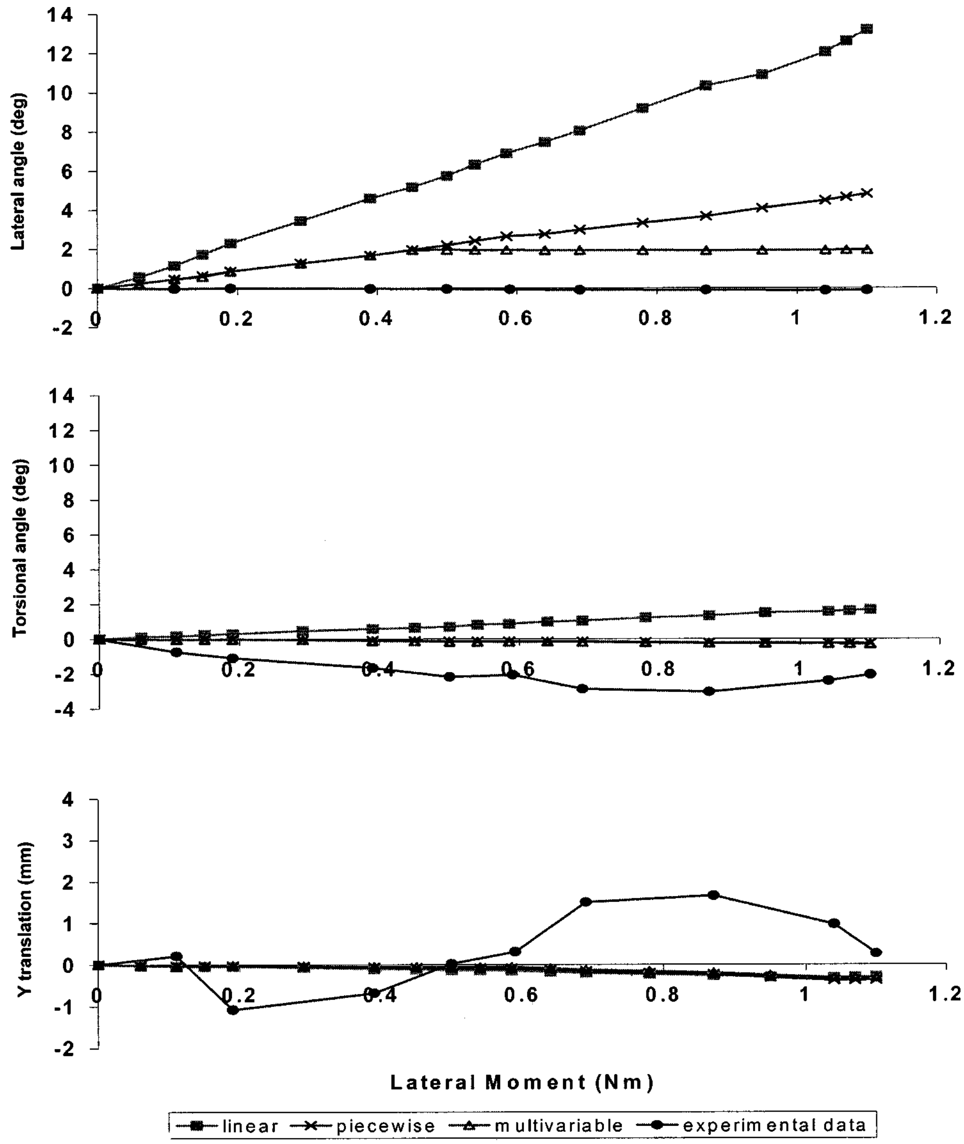

Fig. 6 Lateral displacement components for a typical specimen (same as in Fig. 5) are shown as a function of the applied lateral bending moment from the validation experiment. These plots indicate the failure of the *linear* model to predict the motions out of the sagittal plane well.

ibility testing and the constant definition. Given this sensitivity to the domain of measurement, the errors associated with implementing a single constant tend to be greater than those associated with the piecewise approaches which more closely follow the motion segment's stiffening behavior. Incremental piecewise descrip-

Table 3 Summary of Mean Model RMS Errors of Model Predictions Compared to Experimental Data: RMS errors are shown for each displacement component as predicted by each of the three different model approaches. Data are given as the RMS mean(S.D.).

Component	Linear	Piecewise	Multivariable
Flexion (rad)	0.069 (0.04)	0.027 (0.03)	0.027 (0.02)
Lateral Bending (rad)	0.088 (0.06)	0.058 (0.04)	0.051 (0.04)
Torsion (rad)	0.057 (0.04)	0.044 (0.03)	0.043 (0.03)
Anterior Trans. (mm)	16.25 (45.07)	16.65 (44.91)	16.51 (44.78)
Inferior Trans. (mm)	6.53 (5.78)	5.52 (5.24)	5.58 (5.28)
Lateral Trans. (mm)	2.54 (1.63)	2.33 (1.41)	2.22 (1.43)

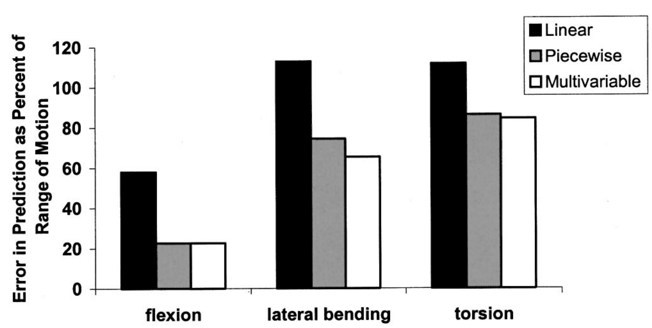

Fig. 7 RMS errors for the different models in their prediction of angular rotations. Here errors are represented as a percentage of the corresponding range of motion in each direction. The similar improvements for the *piecewise* and *multivariable* models over the *linear* one are observed for each motion component.

tions of motion provide more detailed and accurate descriptions and are less sensitive to the experimentalist's ad hoc choices of domain.

Each of the models failed to predict translations well. These translations were, however, small. This occurred because the primary mode of deformation of slender beams is rotation and the local coordinate system origin at the midpoint of the anterior surface of the vertebral body is close to the instant center of rotation reported by others [4,29]. That said, predicted translations contained errors on the order of the measured motion suggesting the need for more accurate tools to predict coupled motions. Applied loads were limited to 2.5 Nm and 50 N in this study to ensure the absence of fatigue damage to the motion segment over the extensive test battery required to generate the model constants and validation data. We also chose one particular initial deformed configuration (i.e. addition of a pretorque) as compared to the neutrally positioned spine. This was chosen because torsion creates a fully three-dimensional loading state [27]. It is possible that in using larger load magnitudes or considering initial positions further from equilibrium that the *multivariable* model might have significantly improved performance over the univariable *piecewise* model.

Structural cervical spine models, as opposed to detailed finite element models, typically serve two functions: prediction of head trajectory and attitude, and prediction of motion segment force and moment. Fortunately, accurate assessment of motion segment translation has little effect on head trajectory or neck load during impact [1]. By contrast, prediction of load within an individual component of the motion segment (e.g. the capsular ligament during whiplash kinematics) may very well depend on the accurate assessment of motion segment translation [30]. As such, improvements in prediction of translation may or may not be required, depending on the intended use of the model. Moreover, further care must be taken when utilizing the flexibility data reported here for describing behavior in more general models of the human neck. For example, in dynamic loading where inertial terms and buckling modes contribute to the observed mechanical responses, the benefit of adding more complex flexibility matrix terms cannot be estimated from this current work.

This study provides previously unavailable three-dimensional human cervical spine motion segment flexibility data. It also examines the effect of varying the complexity of the matrix formulation used to describe motion segment behavior. A *piecewise* nonlinear model of the matrix based on univariable testing was found to offer improved predictive ability over a *linear* model. In contrast, the piecewise nonlinear model with constants derived from *multivariable* testing offers little or no improvement in predictive ability while quickly becoming experimentally intractable.

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